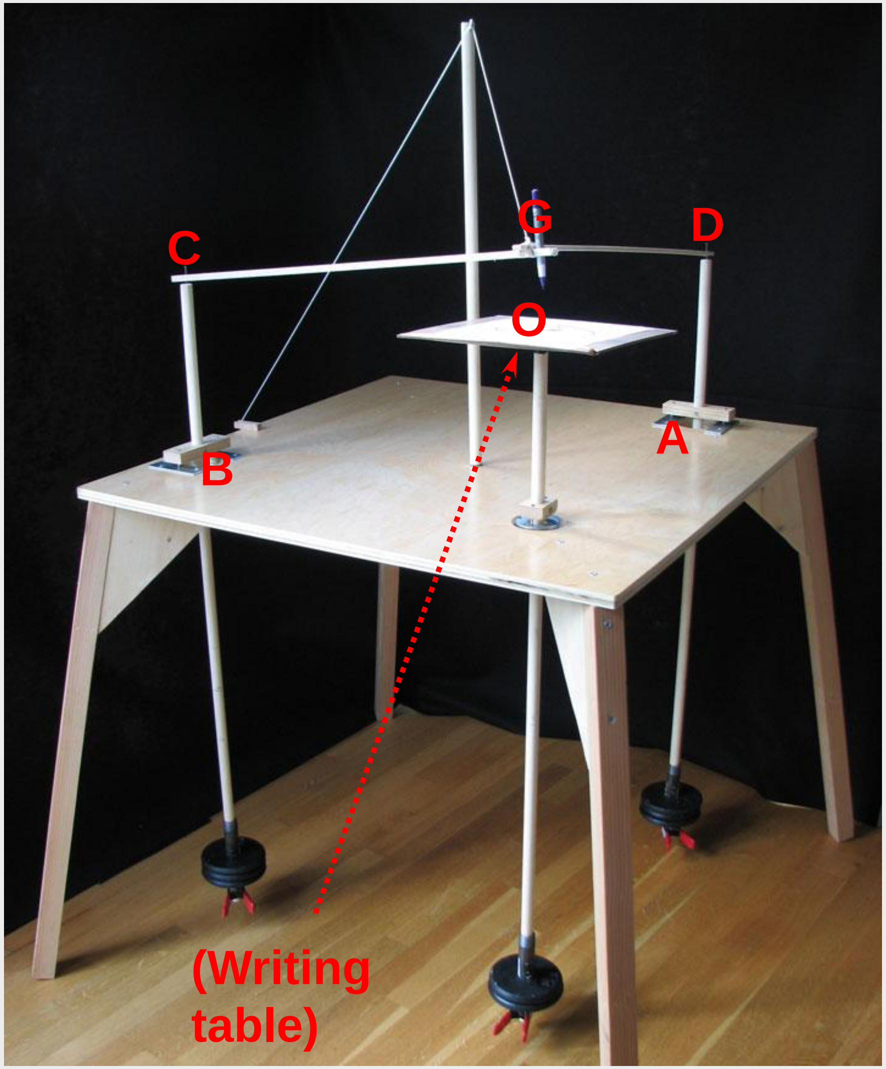
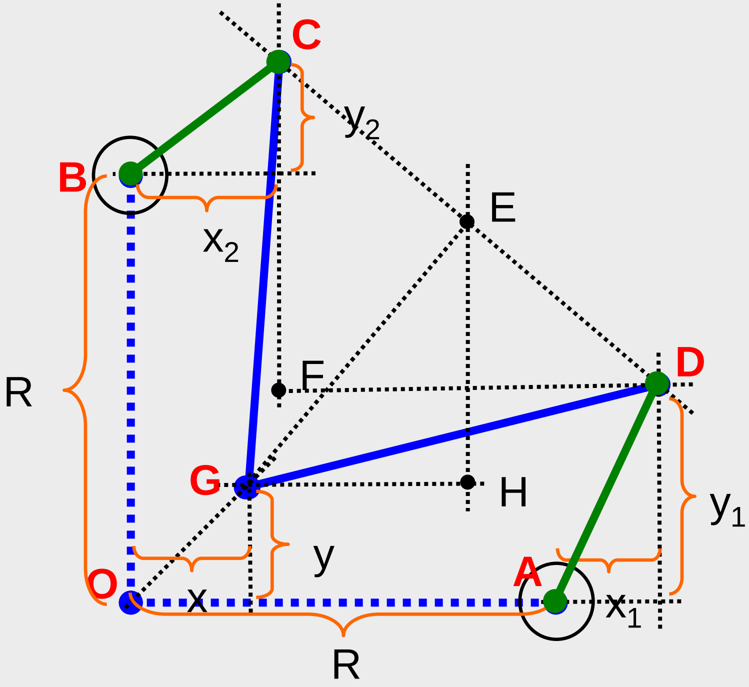
Modeling a Three-Pendulum Harmonograph - Part #1 – by George Lungu

<excelunusual.com>



This first part of the presentation deals with the movement equations of the three pendulums.



Let's start by modeling the movement of the points D, C and the oscillation of the table.

A physical implementation of a harmonograph by Karl Sims
http://www.karlsims.com/harmonograph/index.html

Let's describe the damped oscillation of the end of the ***first pendulum*** *(point D)*

(you can brush-up your knowledge by searching the keyword "Lissajous"):

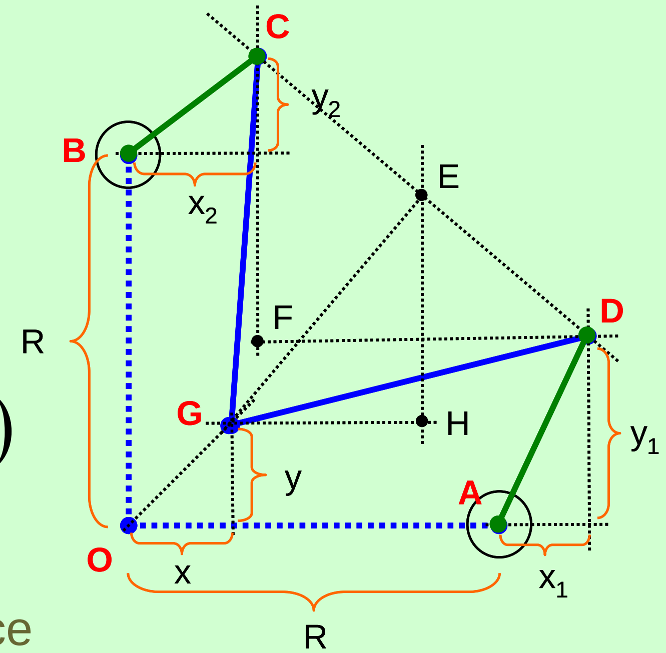
Initial amplitude — Damping factor — Time

$$x_1 = A_1 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_1 \cdot t)$$

$$y_1 = A_1 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_1 \cdot t + \Delta\varphi_1)$$

Damping coefficient — Frequency — Phase difference

Overall Amplitude



- The overall amplitude is not constant but will decrease exponentially in time
- The initial amplitude, damping coefficient, frequency, phase and phase difference are all adjustable
- A phase difference of 0º will result in a straight line oscillation, a 90º in a circle and in an ellipse for any angle in between

Modeling rigid pendulum 2-D oscillations:

- Let's see some examples of how the trajectories might look like. Below there are the oscillation equations of the first pendulum

Initial amplitude

Damping factor

Time

$$x_1 = A_1 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_1 \cdot t)$$

$$y_1 = A_1 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_1 \cdot t + \Delta\varphi_1)$$

Damping coefficient

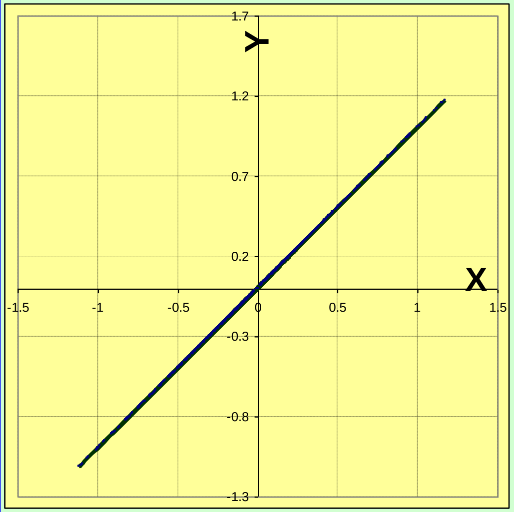
Frequency

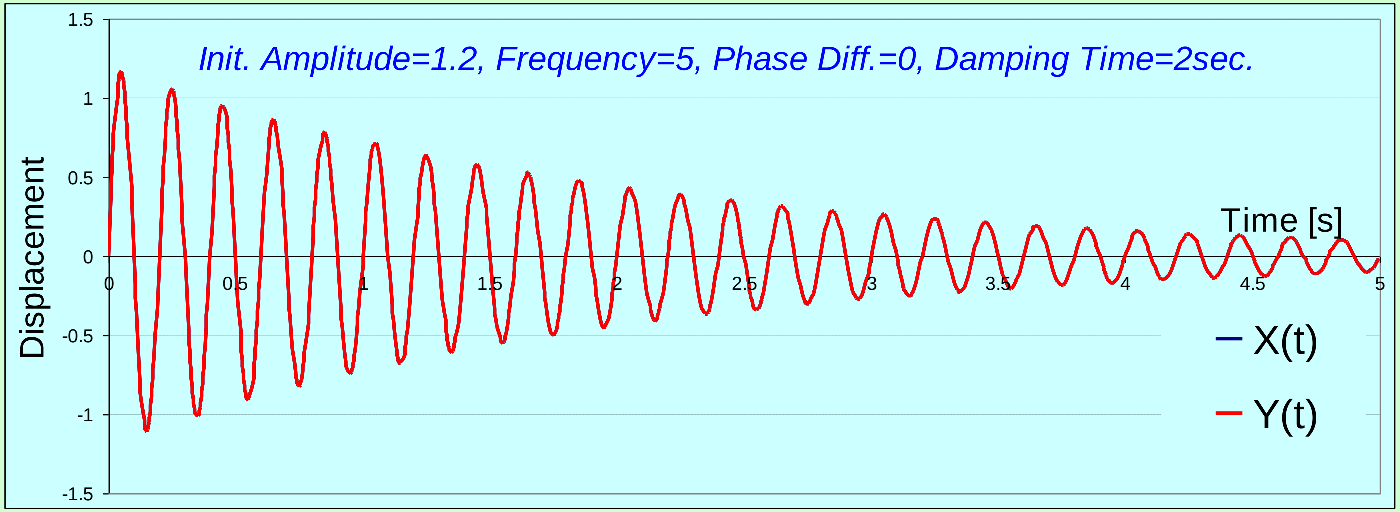
Overall Amplitude

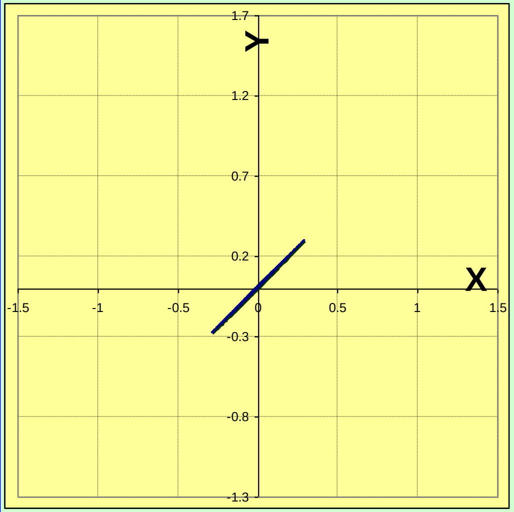
- Insert a worksheet named "Pendulum_Equations" and create a table of data: X(t) and Y(t). After that, we save several parametric plots of data to see the effect of the oscillation parameters on the shape of the pendulum trajectories.

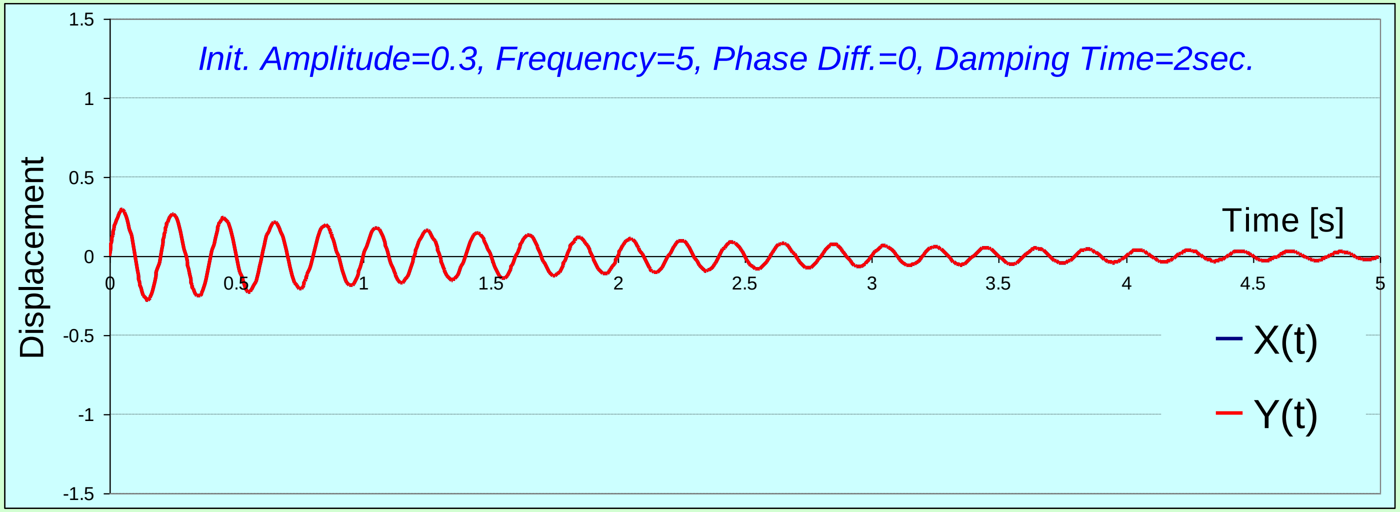
Pendulum Parameter Effects:

- Let’s see several examples of effects of pendulum parameters on pendulum end trajectories:



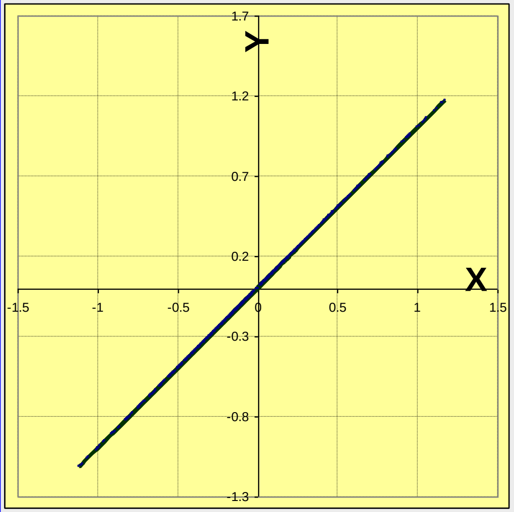


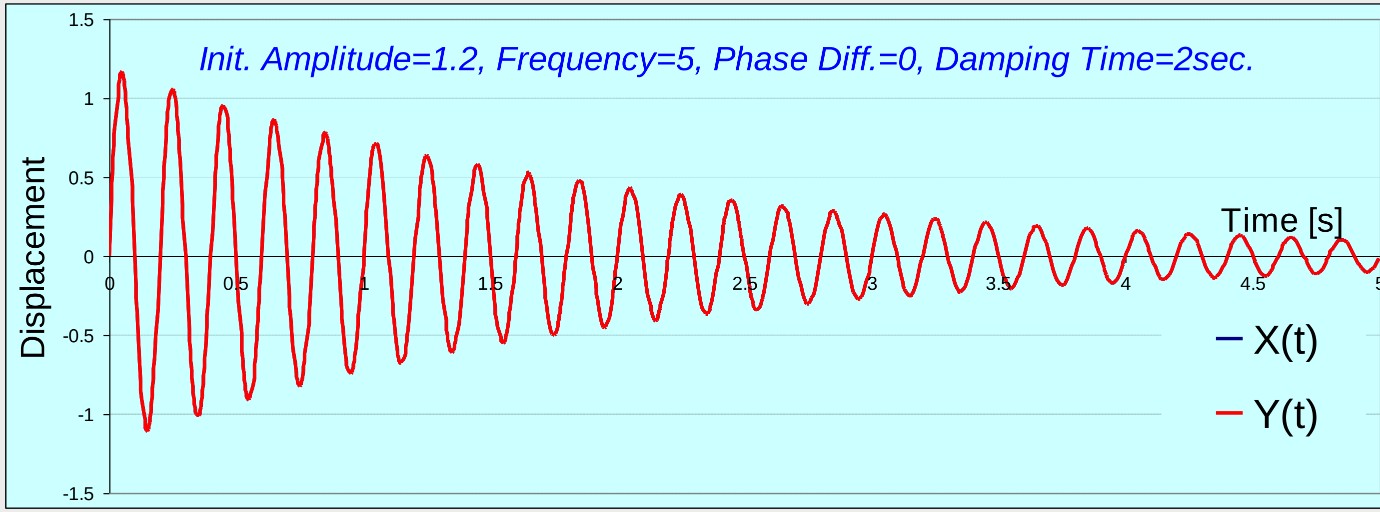


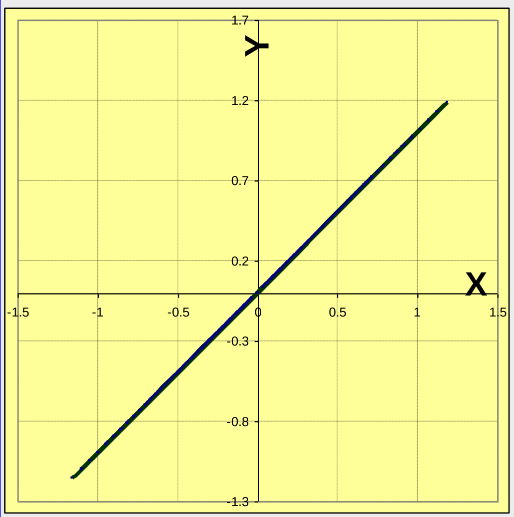


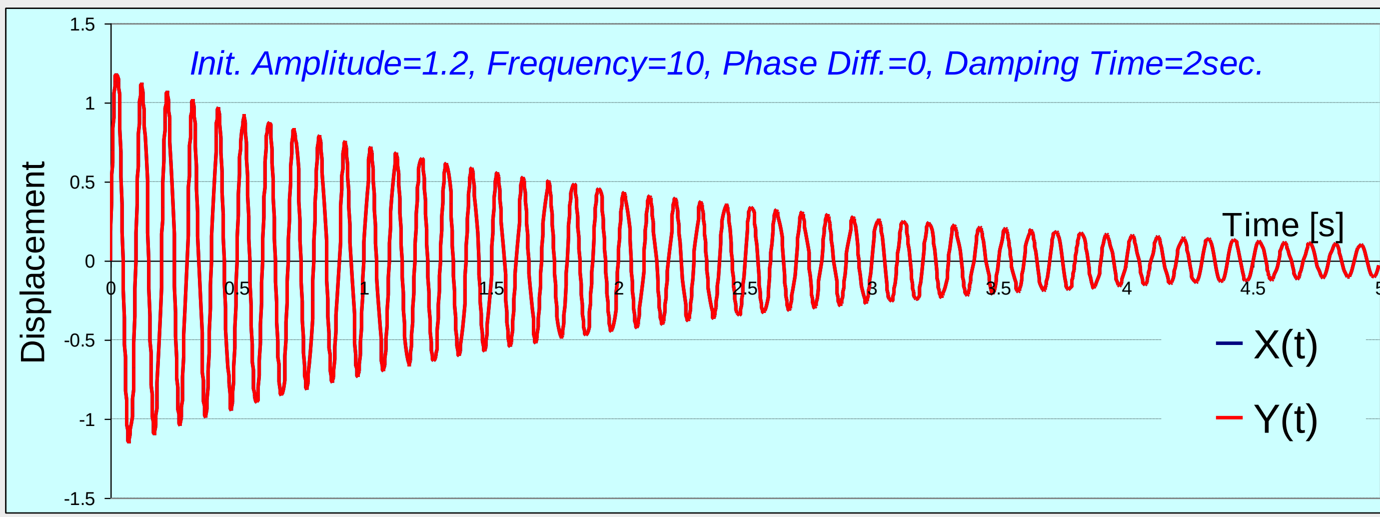
Amplitude change

Pendulum Parameter Effects:



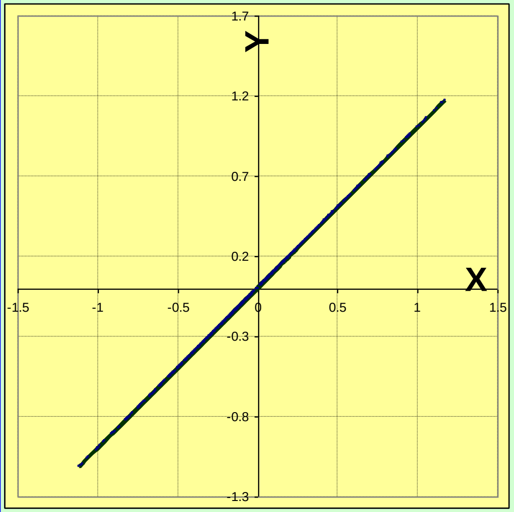


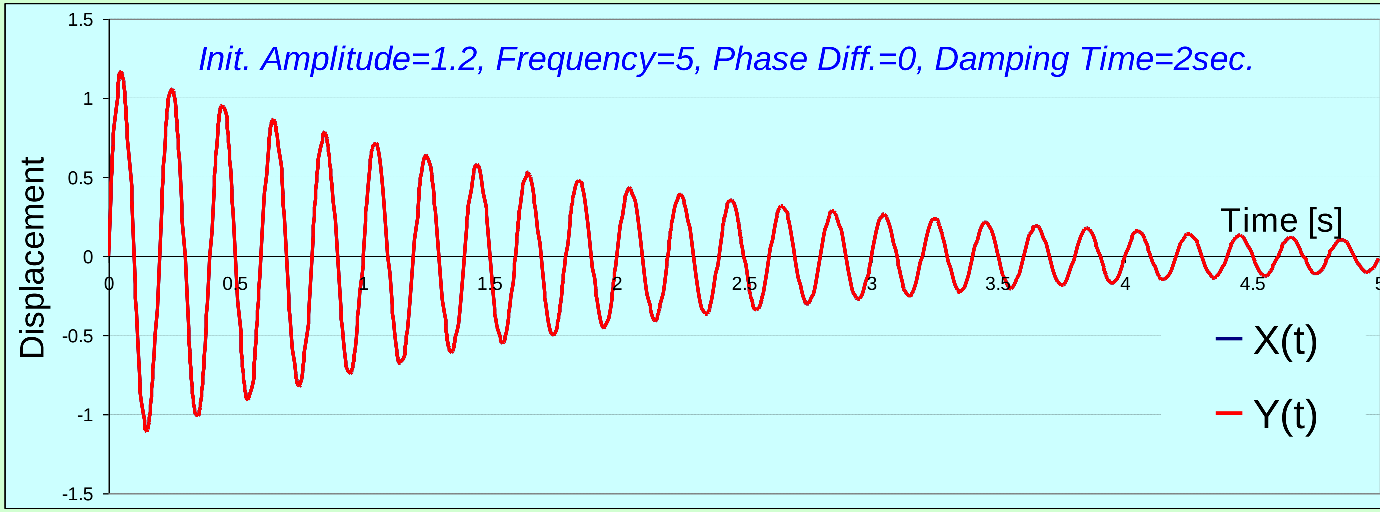


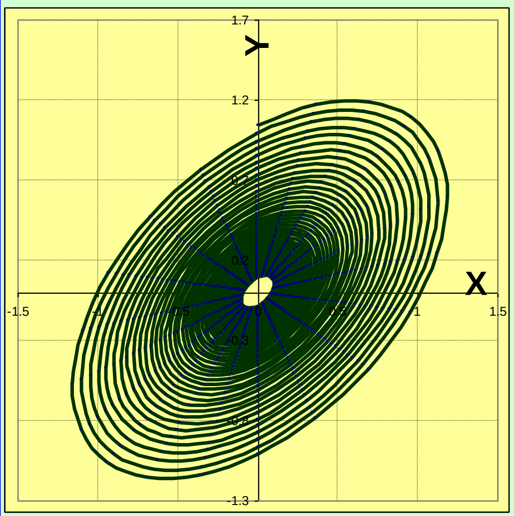


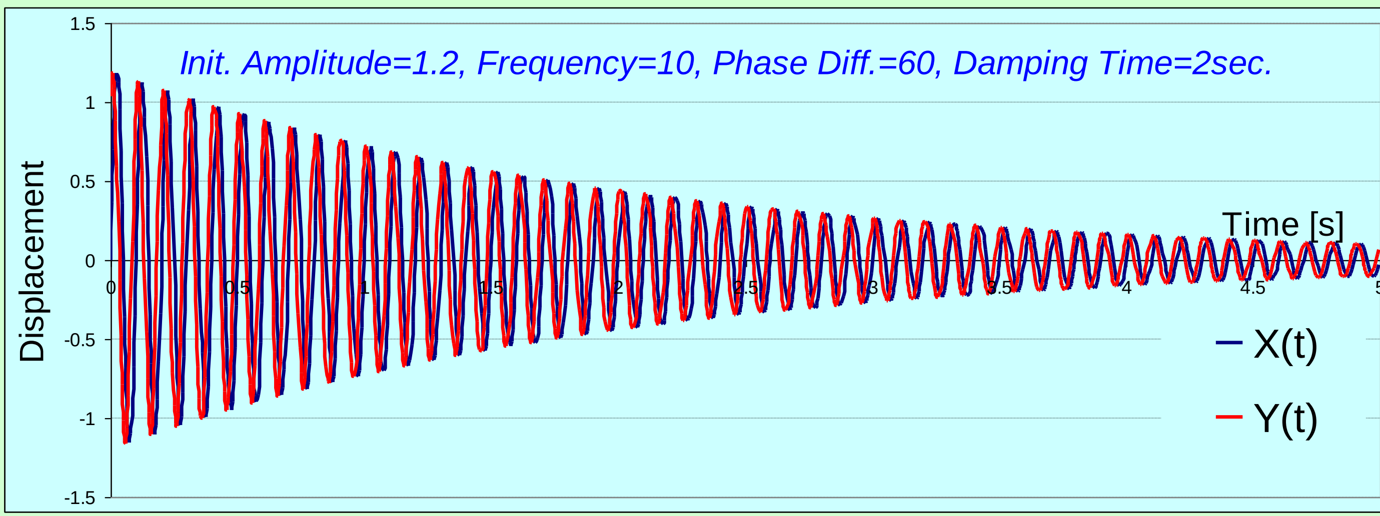
Frequency change

Pendulum Parameter Effects:



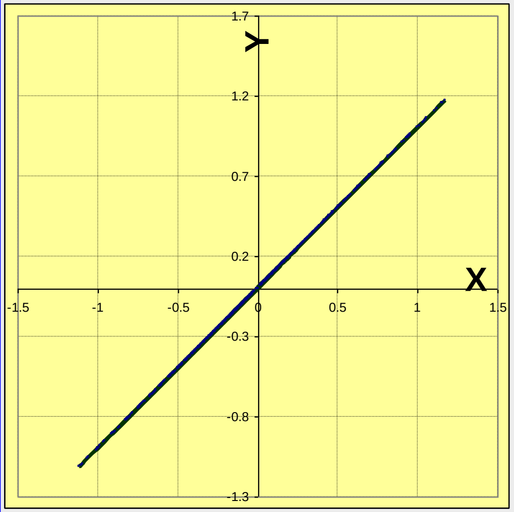


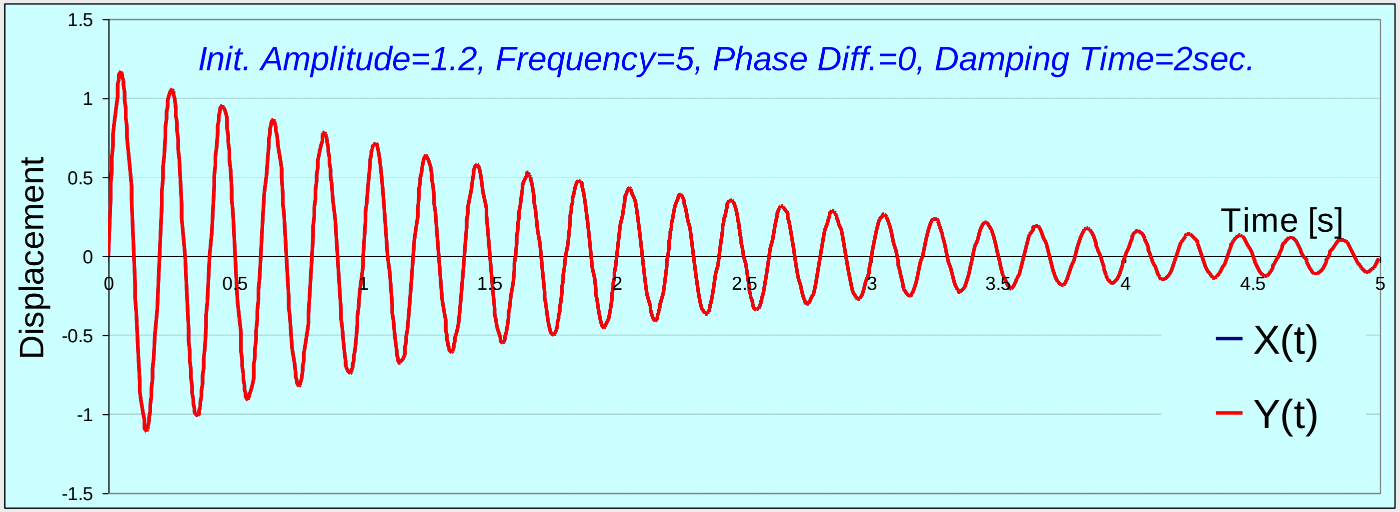


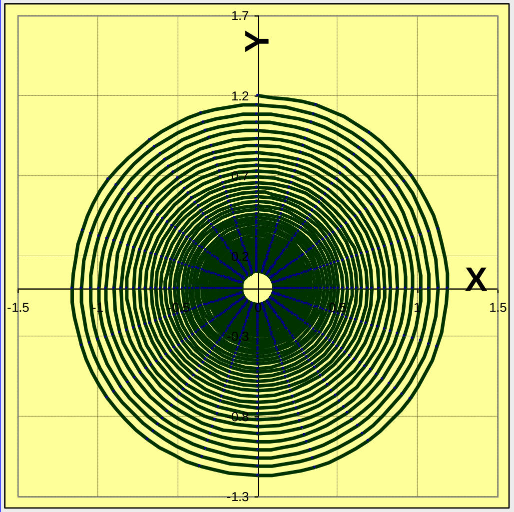


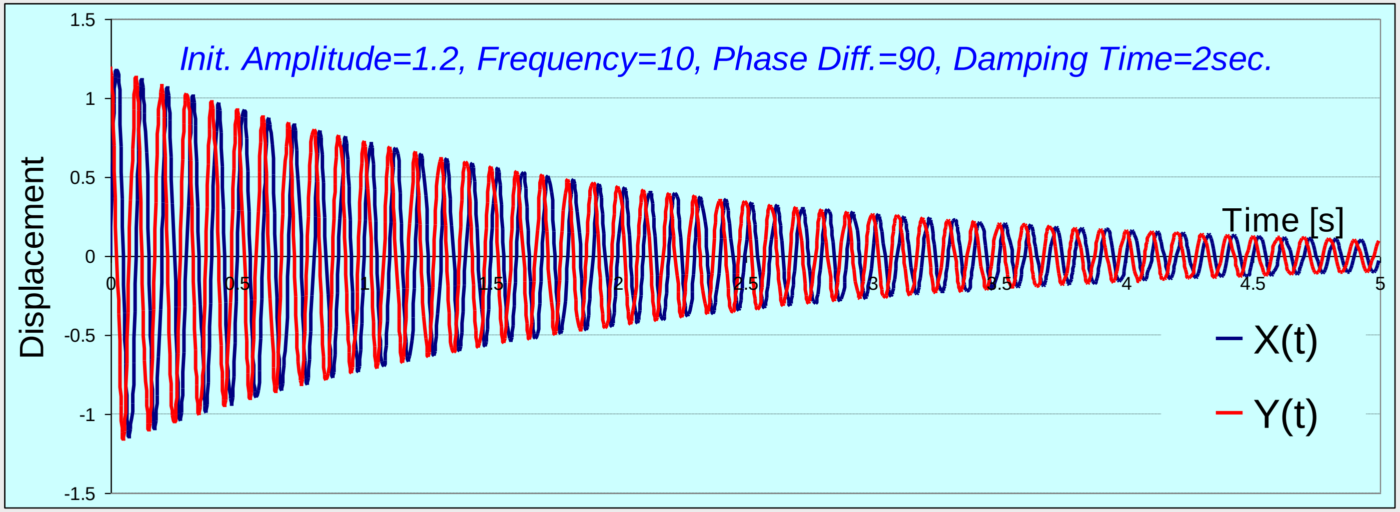
Phase Difference change

Pendulum Parameter Effects:



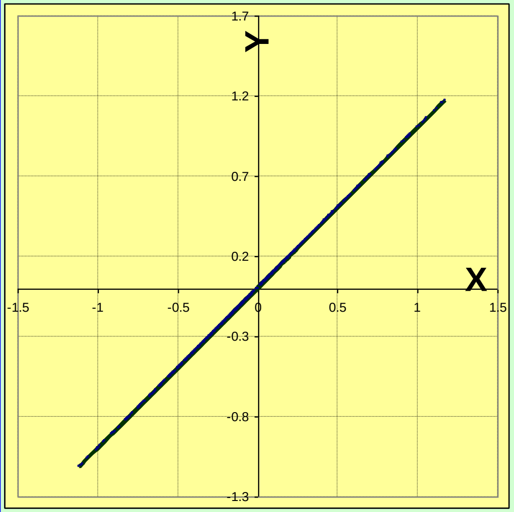


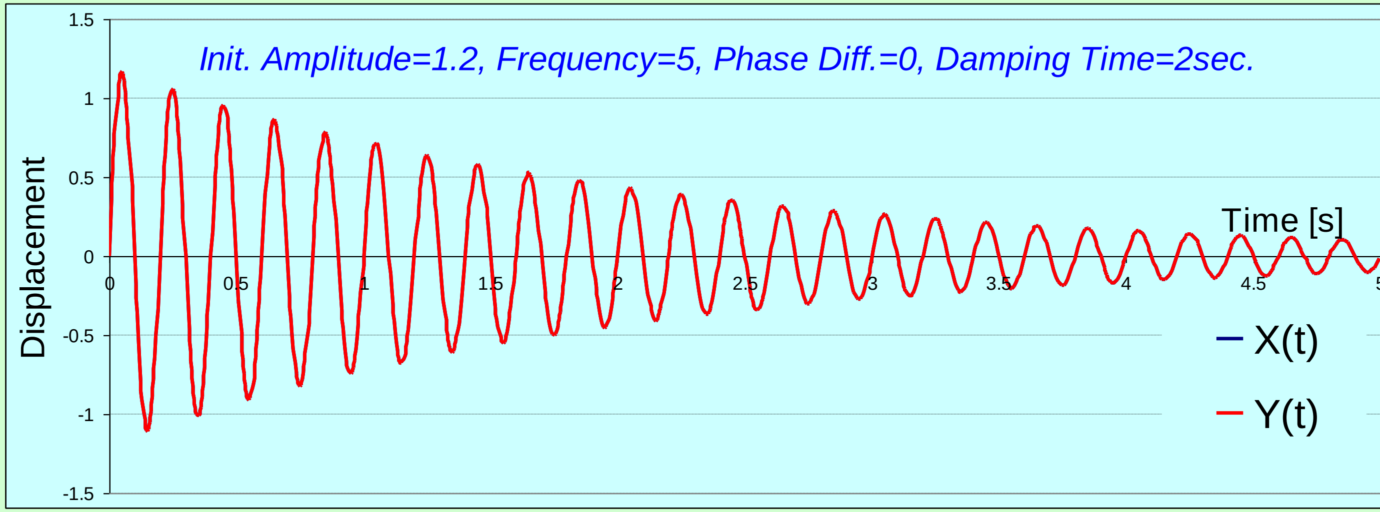


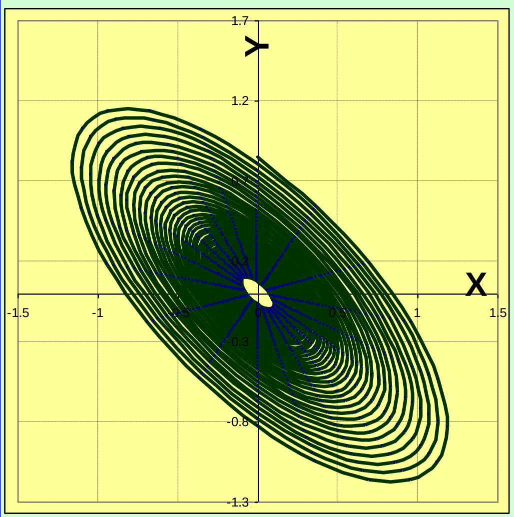


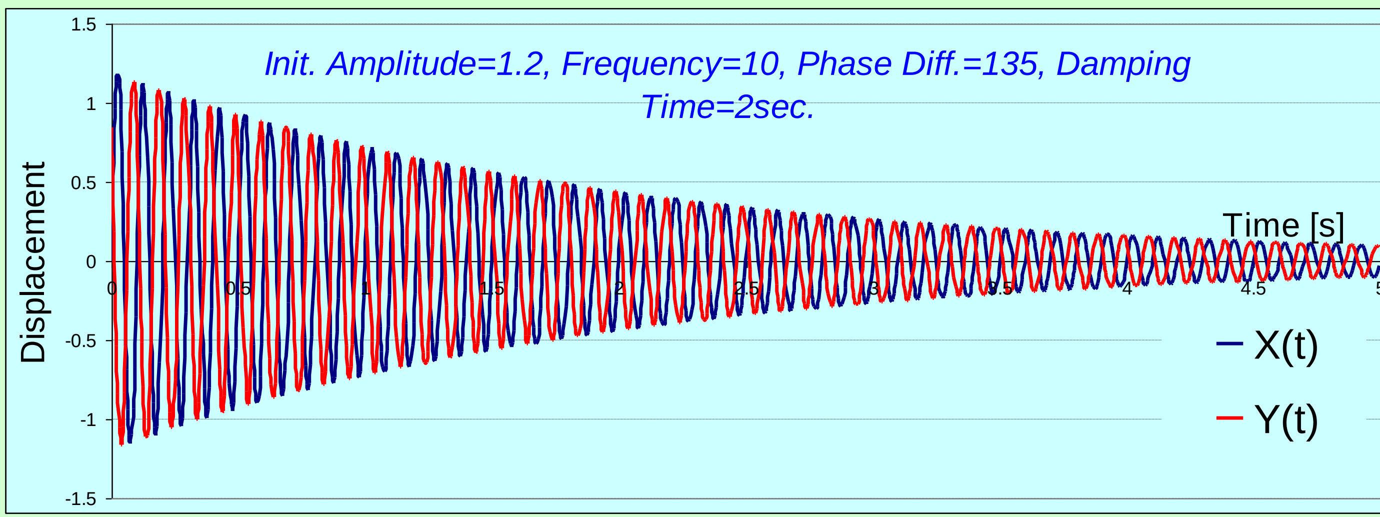
Phase Difference change

Pendulum Parameter Effects:



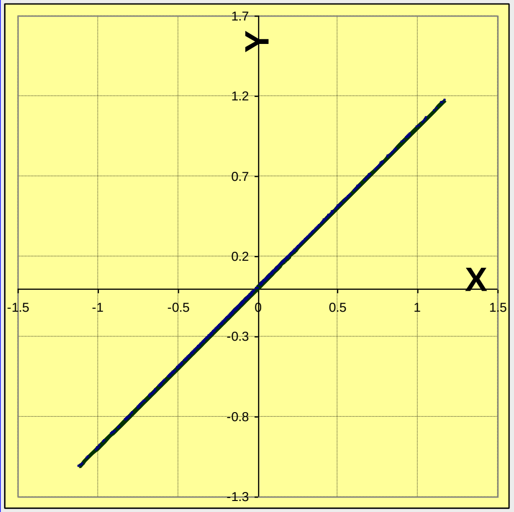


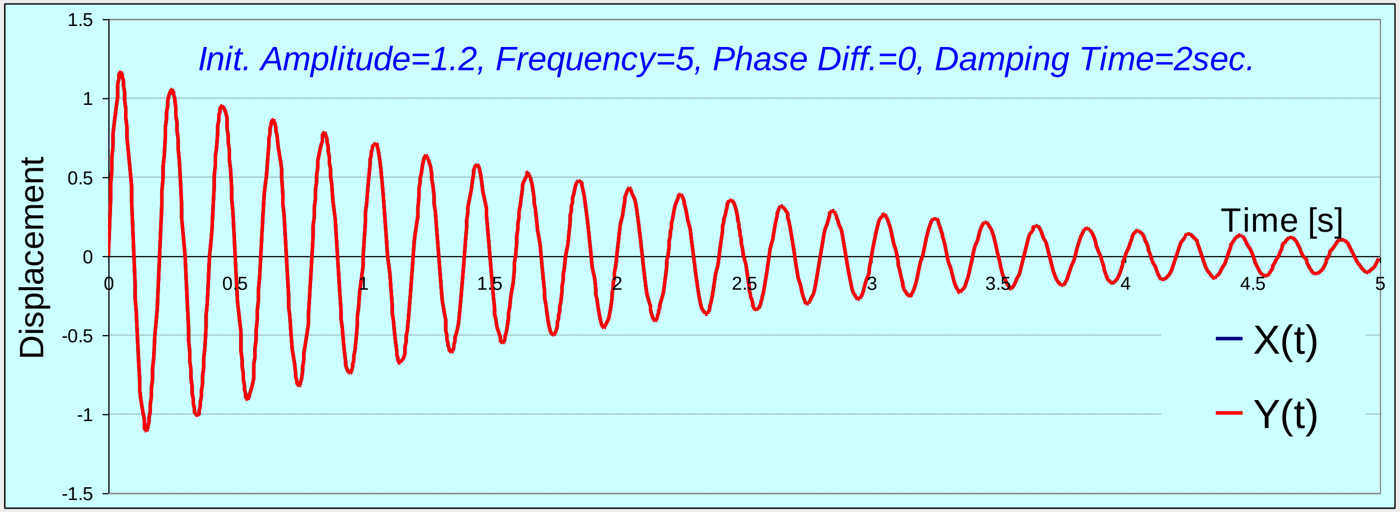


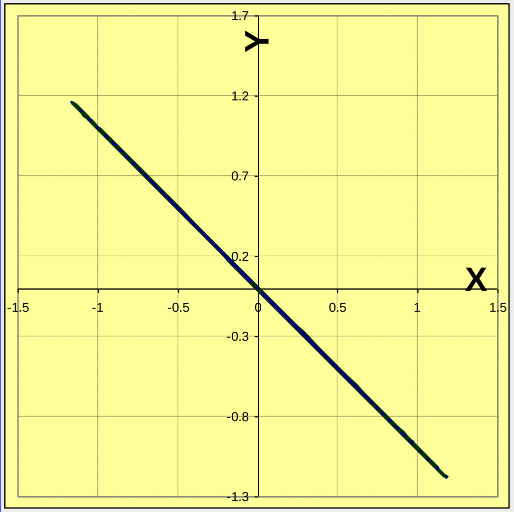


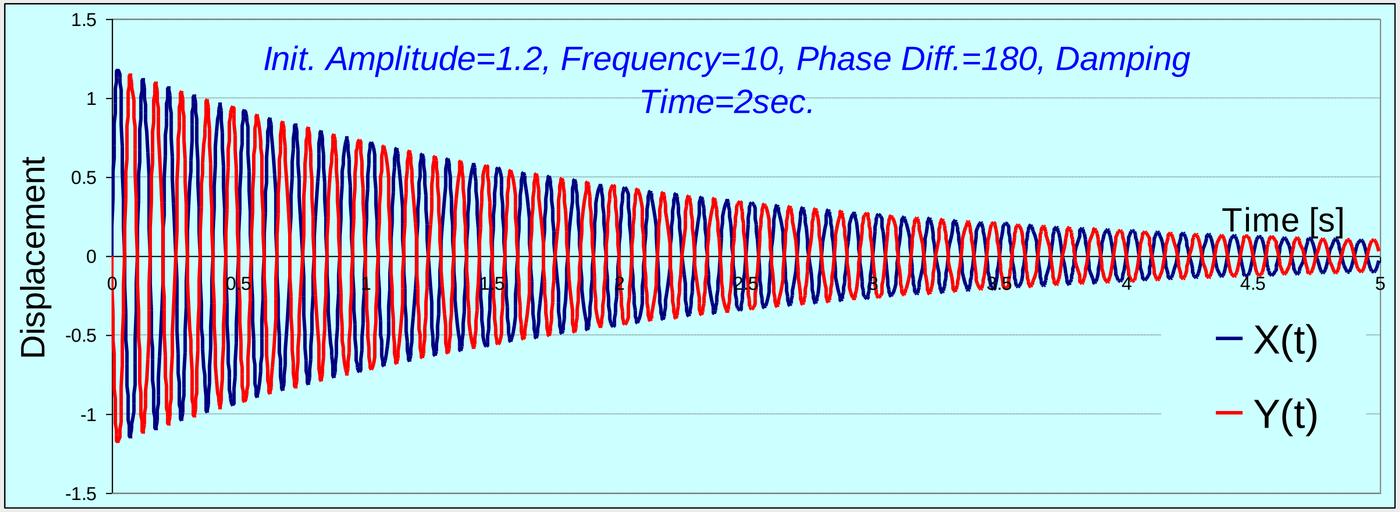
Phase Difference change

Pendulum Parameter Effects:



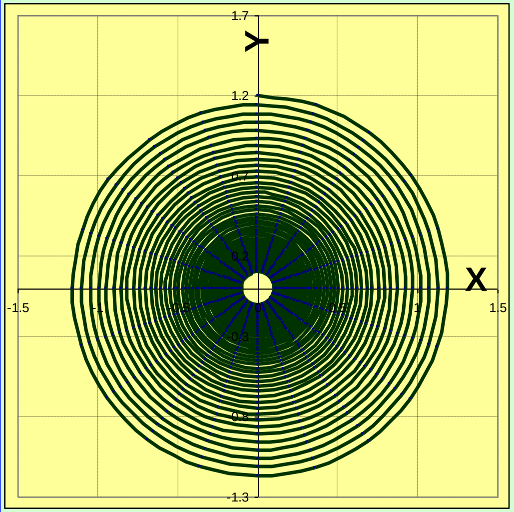


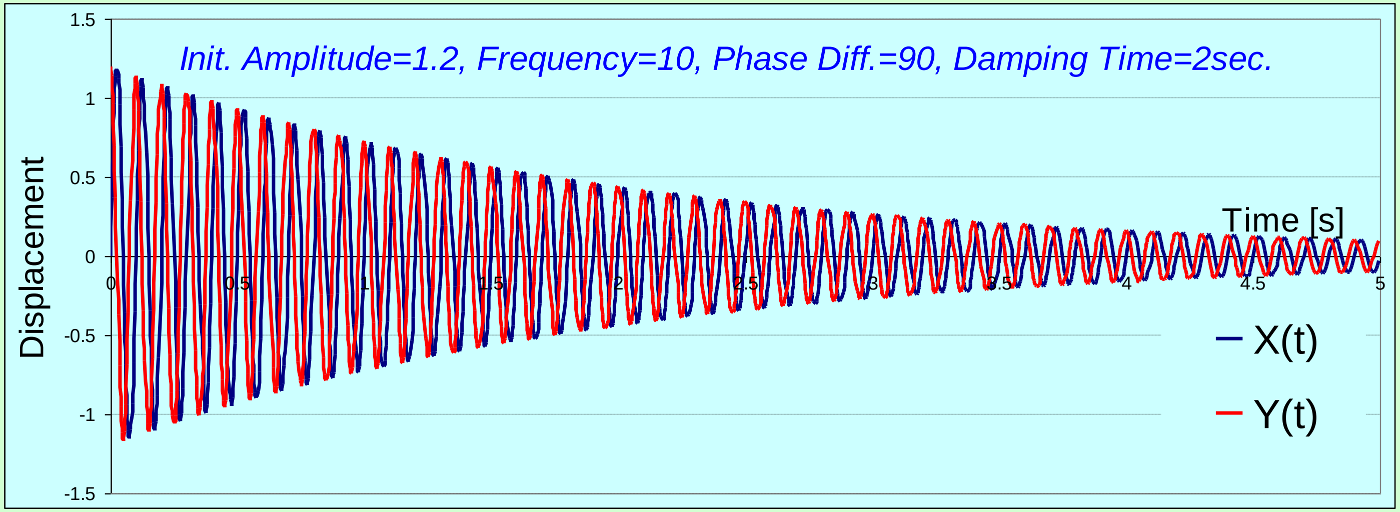


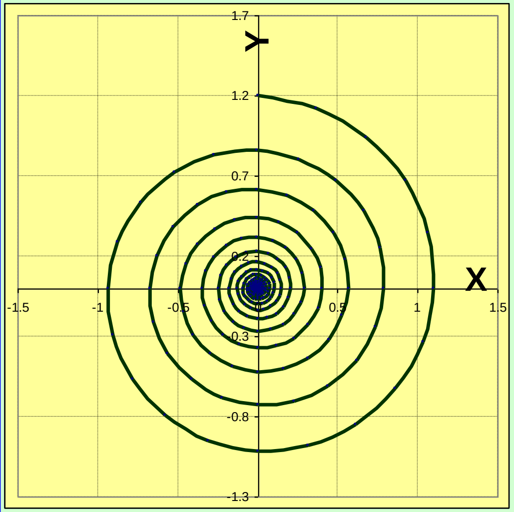


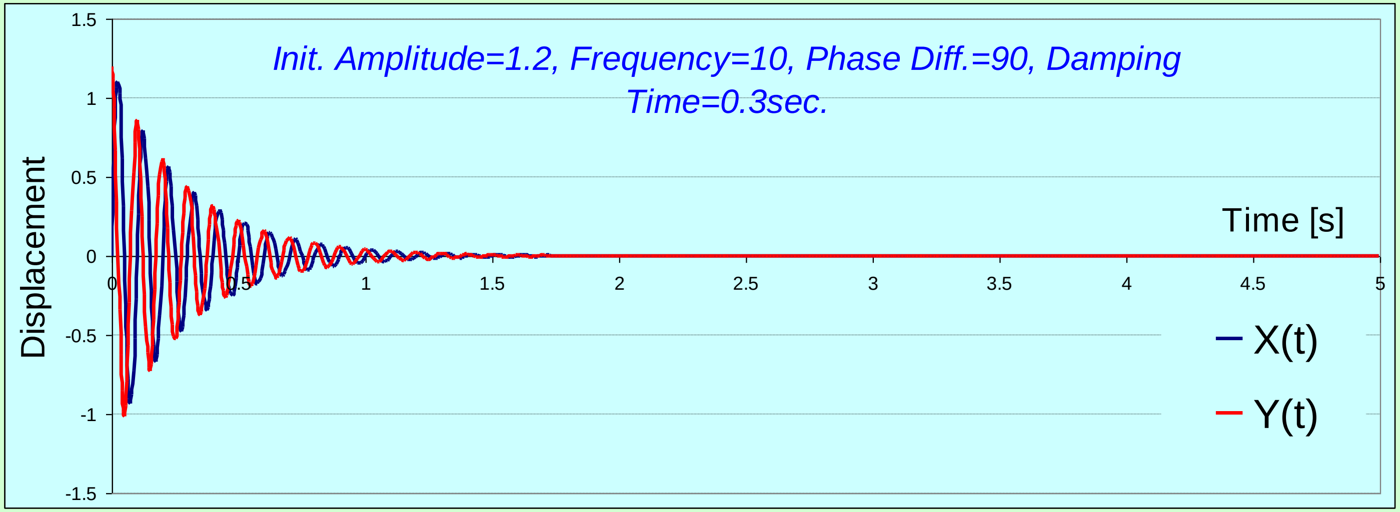
Phase Difference change

Pendulum Parameter Effects:



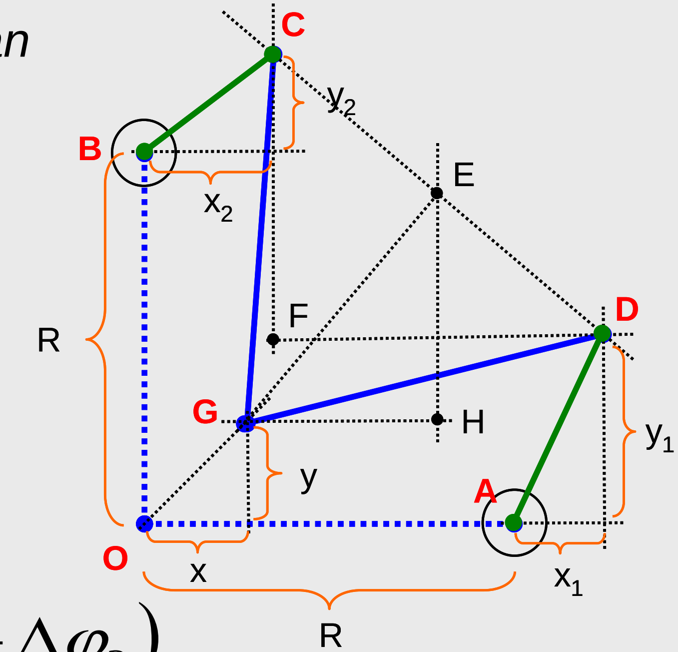






Damping Time change

Coming back to the ***second pendulum***, *(point C) we can write the oscillation equations:*



Initial amplitude — Damping coefficient — Damping factor — Time

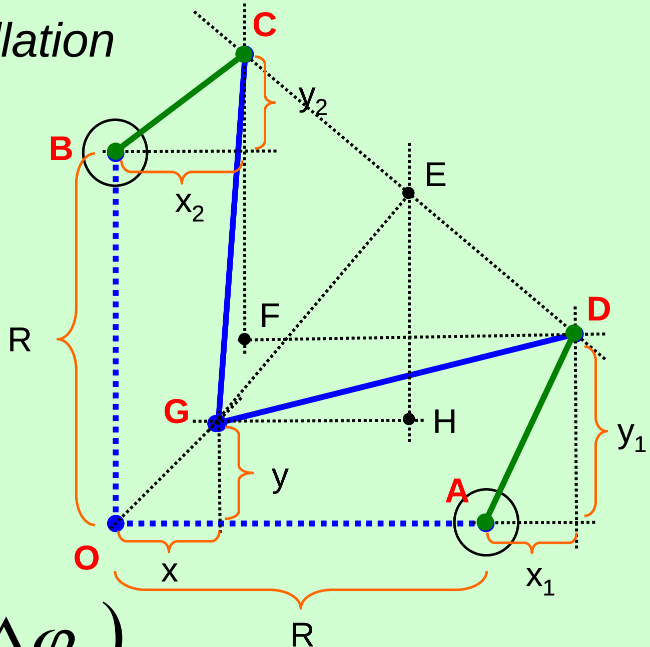
$$x_2 = A_2 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_2 \cdot t + \varphi_2)$$

$$y_2 = A_2 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_2 \cdot t + \varphi_2 + \Delta\varphi_2)$$

Overall Amplitude — Frequency — Phase difference between the second and the first pendulum — Phase difference between x and y

- The overall amplitude is not constant but will decrease exponentially in time
- The initial amplitude, damping coefficient, frequency, phase and phase difference are all adjustable
- A phase difference of 0º will result in a straight line oscillation, a 90º in a circle and in an ellipse for any angle in between

For the ***third pendulum*** *(the table) we can write the oscillation equations:*



Initial amplitude · Damping coefficient · Damping factor · Time

$$x_3 = A_3 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_3 \cdot t + \varphi_3)$$

$$y_3 = A_3 \cdot \exp\left(\frac{-t}{t_{damping}}\right) \cdot \sin(2 \cdot \pi \cdot f_3 \cdot t + \varphi_3 + \Delta\varphi_3)$$

Overall Amplitude

Frequency

Phase difference between the third and the first pendulum

Phase difference between x and y

- The overall amplitude is not constant but will decrease exponentially in time
- The initial amplitude, damping coefficient, frequency, phase and phase difference are all adjustable
- A phase difference of 0^o will result in a straight line oscillation, a 90^o in a circle and in an ellipse for any angle in between